by
Thomas A. Romberg

Mathematical Literacy:

What Does It Mean for School Mathematics?

The Board of Directors of the National Council of Teachers of Mathematics (NCTM) in 1986 established the Commission on Standards for School Mathematics to:

- Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields and
- Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision."[1]

The products of this charge were NCTM's three standards documents published in 1989, 1991,[2] and 1995,[3] and its recently published *Principles and Standards for School Mathematics.*[4]

The Challenge of Literacy

The central tenet underlying this charge is that students will become mathematically literate. The term "literacy" was chosen to emphasize that mathematical knowledge and skills, as defined within the traditional school mathematics curriculum, does not constitute NCTM's primary focus. Instead, the emphasis is on mathematical knowledge put into functional use in a multitude of different situations and contexts in varied, reflective and insight-based ways.
Of course, for such use to be possible and viable, a great deal of fundamental mathematical knowledge and skills are needed.

More broadly the term "literacy" refers to the human use of language.[5] In fact, one's ability to read, write, listen, and speak a language is the most important tool we have through which human social activity is mediated.

Each human language and each human use of language has an intricate design that is tied in complex ways to a variety of functions. For a person to be literate in a language implies that he or she knows many of the design resources of the language and is able to use those resources for several different social functions. Analogously considering mathematics as a language implies that students not only must learn the concepts and procedures of mathematics (its design features), but they must learn to use such ideas to solve non-routine problems and learn to mathematize in a variety of situations (its social functions).

The interplay of "design features" and "functions" can be illustrated by the following example (adapted *from Measuring Student Knowledge and Skills: A New Framework for Assessment*[6]).

The Town Council has decided to construct a streetlight in a small triangular park so that it illuminates the whole park. Where should it be placed? This social problem can be solved following the general strategy used by mathematicians, mathematizing, that can be characterized as having five aspects:

1. Starting with a problem situated in reality (Locating where a street light is to be placed in a park).
2. Organizing it according to mathematical concepts (The park can be represented as a triangle, and equal illumination from a light as a circle with the street light at its center).
3. Gradually trimming away the reality through processes such as making

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assumptions about what are the important features of the problem, generalizing and formalizing (in this case the problem is transformed to locating the center of a circle that circumscribes the triangle).

4. Solving the mathematical problem (Using the fact that the center of a circle that circumscribes a triangle lies at the point of intersection of the perpendicular bisectors of the triangle's sides, construct the perpendicular bisectors of two sides of the triangle. The point of intersection of the bisectors is the center of the circle).
5. And, making sense of the mathematical solution in terms of the real situation (Relating this finding to the real park. Reflecting on this solution and recognizing that if one of the three corners of the park was an obtuse angle, this solution would not be reasonable since the location of the light would be outside the park).

It is these processes that characterize how, in a broad sense, mathematicians *often do mathematics*, how people use mathematics in a variety of current and potential occupations, and how informed and reflective citizens should use mathematics to fully and competently engage with the real world. In fact, learning to *mathematize* should be the primary educational goal for all students.

What Are the Implications?

The implications of teaching for mathematical literacy for schools are many. First, all students need to have the opportunity to learn important mathematics regardless of socio-economic class, gender, and ethnicity. Second, some of the important notions we expect students to learn have changed due to changes in technology and new applications. In fact, technological tools increasingly make it possible to create new, different, and engaging instructional environments. Third, learning to mathematize occurs as a consequence of building on prior knowledge via purposeful engagement in activities and by discourse with other students and teachers in classrooms.

The instructional problem schools face is that since all learning occurs as a consequence of experiences, and all humans have a variety of experiences, virtually all complex ideas in mathematics are understood by a student at a number of different levels in quite different ways. Furthermore, a student's level of understanding will change as a consequence of instructional experiences. Thus, the challenge is how to create classroom experiences so that a student's understanding grows over time. As recently stated in *How People Learn*[7]:

> "Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classrooms."

To accomplish this the pattern of instruction in classrooms needs to become non-routine. Mark Weller in a study of traditional mathematics classrooms found a common routine daily pattern of instruction:

> "It was evident that a repeating pattern of instruction occurred which consisted of three distinctive segments: a review, presentation, and study/assistance period. This 'rhythm of instruction' was not unplanned or coincidental."[8]

Teaching for mathematical literacy constitutes a departure from this traditional daily pattern. For example, Gail Burrill's reflections on this challenge when first teaching a reform unit was as follows:

> "The surprise came when we tried to teach the first lesson. There was little to 'teach'; rather, the students had to read the map, read the keys, read the questions, determine what they were being asked to do, decide which piece of information from the map could be used to help them do this, and finally, decide what mathematics skills they needed, if any, in answering the question. There was no way the teacher could set the stage by demonstrating two examples (one of each kind), or by assigning five 'seat work' problems and then turning students loose on their homework with a model firmly (for the moment) in place."[9]

The changes in the pattern of instruction has forced some teachers to reconsider how they interact with their students. In traditional classrooms how a small number of adults (teachers and others) are able to organize and control a large number of students is seen as a management problem. Management is a particular problem when, as Weller found:

> "… the student[s] … do not necessarily participate willingly in the pursuit of the goals established by the community and the school system" (p. 202).

In this sense the traditional teacher is primarily a manager of resources and personnel, and his or her task is to get students to complete pages or do sets of exercises.

The complexity of instructional issues involved in creating classrooms that

continued on page 8

Mathline Sparks Classroom Changes, Trempealeau Math Teacher Says

In the crook of the Mississippi River lies Trempealeau, a city of 1,000-plus residents. Bluffs, bike trails, and river barges are part of the daily scene outside Trempealeau Middle School walls.

Inside math teacher Bill Schuth oversees a seventh-grade classroom buzzing with activity as students solve problems in cooperative groups. Schuth says this is a dramatic change from the classroom he directed 18 years ago, after arriving in Trempealeau fresh from Winona State University.

Back then, students sat in the traditional five rows and spoke one at a time to the whole class. Now, the activity in Bill Schuth's classroom mirrors what is happening around the state and nation. It's a quiet revolution that is involving all students in the process of learning math.

Schuth says the sparks that initiated classroom changes came from work on his master's degree and from joining Wisconsin Mathline, an online professional development program implemented by the Wisconsin Educational Communications Board. Schuth credits Mathline with having helped him to refine his assessment techniques, teaching strategies, methods of communicating with parents, and discipline policy.

Schuth was a charter member of the Mathline program when it began in 1995. He rejoined last year to participate in the new Algebraic Thinking Math Project, one of four projects in the Mathline program. Each project incorporates PBS-produced video lessons with a facilitated online learning community that includes participants from around the state.

Of the online learning community, Schuth says, "It's great to have people you can go to with questions about curriculum, methods, and motivation. The people online are of such high caliber that you get very reliable input and advice. You often get multiple responses as well."

He adds, "Sometimes teaching can be kind of rough on the spirit; you get more worn out emotionally than physically. It can be a real help to talk to others about your difficulties and frustrations. Many hands *do* make light work, and there are some very willing and able peers available through Mathline."

To illustrate the attitudes and concerns of Mathline learning community members, Schuth talks about recent discussions on the topic of ability grouping. Online participants recognized that students who are not at the top level need to have models of the quick and/or the logical, he says. They stressed that such students need to see others enjoying math in order to realize that math has the potential to be enjoyable. Learning community members also felt strongly that students should be able to follow their own path without finding it blocked by barriers labeled "medium" or "low" ability.

"More that almost anything else that happens in my work day, the [online] dialogs and support of Mathline help me achieve my best as a math teacher," Schuth says. In his small, rural school there is only one other math teacher. The two of them have never had their prep hours at the same time, so his ability to interact with other math teachers was pretty limited.

"When I joined Mathline, I suddenly had 25 other math teachers to converse with on a daily basis. What a resource! The learning community validates and affirms my own perceptions, and I feel more confident about expressing my opinions because they also are held by people I respect."

For more information on participating in the 2001-2002 Mathline program, contact Gina Newell at 608-264-9725 or gnewell@ecb.state.wi.us. Or visit the program's Web site at www.ecb.org/mathline.

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promote mathematical literacy include the interconnected roles of tasks on the one hand and how students and their teachers talk about mathematics on the other; how technological tools can help in the development of classrooms that promote understanding; the normative beliefs within a classroom about how one does mathematics; the organizational structures of the classroom, the role of professional development in helping teachers to develop their own classrooms that promote understanding; how the school, as an organization, supports (or impedes) the work of teachers in developing and sustaining these classrooms; and how non-school agents (such as parents), agencies (district), and their actions support (or impede) the development of these classrooms.

Impediments to Change

The fact is that classrooms that focus on a non-routine pattern of instruction that allows students to become mathematically literate are not easy to create. Nevertheless, it is possible, and with appropriate guidance from teachers, students can learn to mathematize. Unfortunately, the problem with the vision of school mathematics, as outlined in the previous paragraphs, is that they are ideas put forward by educational leaders, policymakers, and professors about what mathematical content, pedagogy, and assessments should be. Implementation of such ideals can be undermined by a number of factors. For example, not everyone agrees with the goal of mathematical literacy for all; some influential persons believe that the traditional course of study for school mathematics works reasonably well.

In fact, as Labaree points out, during the past century calls for reform have had "remarkably little effect on the character of teaching and learning in American classrooms."[10]

In conventional classrooms the mathematical content is cut off from practical problem situations and taught in isolation from other subjects, students are differentiated by ability and sequenced by age, and instruction is grounded in textbooks and delivered in a teacher-centered environment.

Instead of changing conventional practices the common response to calls for reform has been the "nominal" adoption of the reform ideas. Schools have used the reform labels but did not follow most of the practices advocated.[11] Thus, to document the impact of any reform efforts in classrooms one needs to examine the degree to which the reform vision actually has been implemented.

Finally, to be consistent with the standards-based vision the quality of student performance should be judged in terms of whether students are mathematically literate. This means information should be gathered about what concepts and procedures students know with understanding and how students can use such knowledge to mathematize a variety of non-routine problem situations. Only then can one judge whether student performance meets the reform vision, and in turn whether the changes meet society's needs.

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Unfortunately the existing instruments commonly used to judge student performances in mathematics were not designed to assess mathematical literacy. Standardized tests used by school districts in the United States measure the number of correct answers a student can give to questions about knowledge of facts, representing and recognizing equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, manipulating expressions containing symbols and formulae in standard form, and doing calculations. At best these tests measure a student's knowledge of some of the "design features" associated with mathematical literacy. Also, it is questionable as to whether such instruments measure understanding of such features. And none makes any serious attempt to assess student capability to mathematize.

Thus, to assess the intended impact of standards-based reforms in mathematics a new assessment system will need to be developed.

The current rhetoric about the need to reform the teaching and learning of mathematics is based on NCTM's vision of what it means to be mathematically literate. This notion implies that students not only must learn the concepts and procedures of mathematics, they must learn to uses such ideas to solve non-routine problems. The implications of this vision on schooling practices involve shifting from routine to non-routine instruction and using assessments aligned with the mathematical literacy. ◗

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continued on page 31

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Ken Cole, Executive Editor.

continued from page 8

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Romberg is a former director of the National Center for Improving Student Learning and Achievement in Mathematics and Science at the Wisconsin Center for Education Research and a professor of curriculum and instruction (mathematics education). His research has focused on young children's learning of initial mathematical concepts, methods of evaluating students and programs, and integrating research on teaching, curriculum, and student thinking. In 1997 he received the National Council of Teachers of Mathematics' Lifetime Achievement Medal in Leadership, Teaching, and Service.